PROF. RAJENDRA SINGH (RAJJU BHAIYA) UNIVERSITY, PRAYAGRAJ

PROPOSED STRUCTURE

AND

DETAILED SYLLABUS

FOR

PROGRAM: M.A./M.Sc.

SUBJECT: MATHEMATICS

Session: 2022-2023 onwards

PROF. RAJENDRA SINGH (RAJJU BHAIYA) UNIVERSITY, PRAYAGRAJ
Structure of Syllabus
Program: M.A./M.Sc. Subject: Mathematics

Structure of Syllabus Developed by			
Name of BoS Convener/BoS Member	**Designation**	**Department**	**College/ University**
Prof. Archana Sinha	Professor & Convener		K. A. P.G. College, Prayagraj
Prof. P.K. Singh	Professor & Subject Expert- Member	Department of Mathematics	University of Allahabad
Dr. B. K. Sharma	Assistant Professor & Subject Expert- Member	Department of Mathematics	University of Allahabad
Dr. Sapna Devi	Assistant Professor & Subject Expert- Member	Department of Mathematics	University of Allahabad
Dr. Iftekhar Ahamad Ansari	Assistant Professor & Member	Department of Mathematics	H.N.B Govt. P. G. College, Prayagraj
Dr. Rajendra Prasad	Assistant Professor & Member	Department of Mathematics	Govt. P. G. College, Saidabad, Prayagraj

Course Code		**Course Title**	**Credits**	**T/P**	**Evaluation**	
					CIE	**ETE**
A	B	C	D	E	F	G
SEMESTER I (YEAR I)						
B030701T	CORE	Algebra I	5	T	25	75
B030702T	CORE	Analysis	5	T	25	75
B030703T	CORE	Differential Geometry	5	T	25	75
B030704T	FIRST ELECTIVE (Select any one)	Partial Differential Equations and Integral Equations	5	T	25	75
B030705T		Basic Number Theory				
B030706P	SECOND ELECTIVE (Select any one)	Project Presentation	4	P	50	50
B030707P		Computational Techniques using C				
SEMESTER II (YEAR I)						
B030801T	CORE	Algebra II	5	T	25	75
B030802T	CORE	Topology	5	T	25	75
B030803T	CORE	Classical Mechanics	5	T	25	75

B030804T	THIRD ELECTIVE (Select any one)	Tensors and Riemannian Geometry	5	T	25	75
B030805T		Hydrodynamics				
B030806P	FOURTH ELECTIVE (Select any one)	Project Presentation	4	P	50	50
B030807P		Introduction to Latex				
SEMESTER III (YEAR II)						
B030901T	CORE	Measure and Integration	5	T	25	75
B030902T	CORE	Ordinary Differential Equations and Boundary Value Problems	5	T	25	75
B030903T	CORE	Differential Geometry of Manifolds	5	T	25	75
B030904T	FIFTH ELECTIVE (Select any one)	Mathematical Modelling	5	T	25	75
B030905T		Advanced Linear Algebra				
B030906P	SIXTH ELECTIVE (Select any one)	Project presentation based on Survey/Seminar/Assignment	4	P	50	50
B030907P		Introduction to SCILAB/ MATLAB				
SEMESTER IV (YEAR II)						
B031001T	CORE	Functional Analysis	5	T	25	75
B031002T	CORE	Advanced Fluid Mechanics	5	T	25	75
B031003T	SEVENTH ELECTIVE (Select any one)	Wavelets	4	T	25	75
B031004T		Representation Theory of Finite Groups				
B031005T		Algebraic Number Theory				
B031006T		Special Functions				
B031007T		Galois Theory				
B031008R	RESEARCH PROJECT/ DISSERTATION	Major Research Project/ Dissertation	10	R	50	50

PROF. RAJENDRA SINGH (RAJJU BHAIYA) UNIVERSITY, PRAYAGRAJ

Detailed Syllabus for

Program: M.A./M.Sc. Subject: MATHEMATICS
(w.e.f. 2022-23)

M.A./M.Sc. SEMESTER I (YEAR I)

PAPER-I: ALGEBRA-I

Unit-I: Life and Contributions of Srinivasa Ramanujan.

Isomorphism theorems for groups, Symmetric and alternating groups, Dihedral groups, Normal subgroups of Sn and A_n, Internal and external direct products of groups, Indecomposable groups.

Unit-II: Action of a group on a set, Stabilizer subgroups and orbit decomposition, core of a subgroup, class equation of an action, Sylow's first, second and third theorems, Applications of Sylow's theorems, Groups of order pq.

Unit-III: Composition series of a group, Commutator series of groups, Solvable groups, Solvability of finite p-groups, Upper and lower central series of groups, Nilpotent groups, Structure theorem of finite abelian groups.

Unit-IV: Factorization in integral Domains, Primes and irreducible elements, Euclidean domains, Principal ideal domains, Unique factorization domains, gcd, Polynomial rings over domains, Chinese remainder theorem, Eisenstein's irreducibility criterion, Unique factorization in polynomial rings over UFD.

Unit-V: Modules over ring, Submodules, Module homomorphisms and quotient modules, isomorphism theorems and correspondence theorem, Internal and external direct sums, Exact sequences, Split exact sequences, Five lemma.

Books Recommended:

1. Ramji Lal, Algebra I, Infosys Foundation Series in Mathematical Sciences, Springer, 2017.
2. V. Sahai and V. Bist, Algebra, Narosa Publishing House, 2008.
3. D. S. Dummit and R. M. Foote, Abstract Algebra, Wiley, 2002.
4. J. A. Gallian, Contemporary Abstract Algebra, Cengage India Pv. Ltd., 2019
5. T. W. Hungerford, Algebra, Springer, 1974.
6. R. Kanigel, The Man Who Know Infinity, C. Scribner's, 1991.
7. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-II: ANALYSIS

Unit-I: Equivalent sets, Countable and uncountable sets, Uncountability of $\mathbb{R}$ and P(N), Cardinality and cardinal arithmetic, Schröder-Bernstein theorem, Euclidean space $\mathbb{R}^n$, Structure of open sets in $\mathbb{R}$, Bolzano-Weierstrass theorem, Cantor intersection theorem.

Unit-II: Pointwise and uniform convergences of sequence and series of functions, Cauchy's criterion for uniform convergence, Weierstrass M-test, Abel's and Dirichlet's tests for uniform convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, Statements of Weierstrass approximation theorem and Abel's limit theorem.

Unit-III: Partial derivatives, direcional derivatives and total derivatives of functions of several real variables, Chain rule, Mean value theorems, Taylor's theorems, Jacobian, Inverse function theorem and Implicit function Theorem.

Unit-IV: Evaluation of improper integral and integrals of trigonometric functions by contour integration, Zeros of analytic functions, Identity theorem for analytic functions, Maximum modulus theorem, Schwarz' lemma, Argument principle, Rouche's theorem, Open mapping theorem,

Unit-V: Hurwitz's Theorem, Infinite products, Weierstrass' factorization theorem, Mittag-Leffler's Theorem, Gamma function and its properties, The Riemann zeta function, Order and genus of entire functions, Hadamard's factorization theorem.

Books Recommended:

1. Rudin, W., Principles of Mathematical Analysis, 3rd ed., McGraw-Hill, 1983.
2. Apostol, T., Mathematical Analysis, 2nd ed., Narosa Publishing House , 2002.
3. Ponnusamy, S., Foundations of Complex Analysis, 2nd ed., Narosa Publishing House, 2005.
4. Conway, J. B. , Functions of One Complex Variable, 2nd ed., Narosa Publishing House, 2000.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-III: DIFFERENTIAL GEOMETRY

UNIT I: Curves in space R^3, parameterized curves, regular curves, helices, arc length, reparametrization (by arc length), tangent, principal normal, binormal, osculating plane, normal plane, rectifying plane, curvature and torsion of smooth curves, Frenet-Serret formulae, Frenet approximation of a space curve.

UNIT II: Osculating circle, osculating sphere, spherical indicatrices, involutes and evolutes, intrinsic equations of space curves, isometries of R^3, fundamental theorem of space curves, surfaces in R^3, regular surfaces, co-ordinate neighborhoods, parameterized surfaces, change of parameters, level sets of smooth functions on R^3, surfaces of revolution, tangent vectors, tangent plane, differential of a map.

UNIT III: Normal fields and orientability of surfaces, angle between two intersecting curves on a surface, Gauss map and its properties, Weingarten map, second and third fundamental forms, classification of points on a surface.

UNIT IV: Curvature of curves on surfaces, normal curvature, Meusnier theorem, principal curvatures, geometric interpretation of principal curvatures, Euler theorem, mean curvature, lines of curvature, umbilical points, minimal surfaces, definition and examples, Gaussian curvature, intrinsic formulae for the Gaussian curvature, isometries of surfaces, Gauss Theorem Egregium (statement only).

UNIT V: Christoffel symbols, Gauss formulae, Weingarten formulae, Gauss equations, Codazzi-Mainardi equations, curvature tensor, geodesics, geodesics on a surface of evolution, geodesic curvature of a curve, Gauss-Bonnet Theorem (statement only).

Books Recommended:

1. M. P. Do Carmo, Differential Geometry of Curves and Surfaces, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1976.
2. B. O' Neill, Elementary Differential Geometry, Academic Press, 1997.
3. A. Pressley, Elementary Differential Geometry, Springer (Undergraduate Mathematics Series), 2001.
4. D. Somasundaram, Differential Geometry, A First Course, Narosa Publishing House, New Delhi, 2005.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-IV: Any one of the following :

PAPER-IV(a): PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS

Unit-I: Preliminaries of PDE's, Linear PDE's with constant coefficients, Reduction to canonical forms, Classification of second order P.D.E.'s.

Unit-II: Method of separation of variables: Laplace, Diffusion and Wave equations in Cartesian, cylindrical and spherical polar coordinates, D'Alembert and Riemann-Volterra solutions of one-dimensional wave equation.

Unit-III: Boundary value problems for transverse vibrations of strings and heat diffusion in a finite rod, Non-linear PDE's of second order: Monge's method of solution.

Unit-IV: Classification of linear integral equations: Volterra and Fredholm integral equations, Relation between differential and integral equations,Conversion of initial and boundary value problems into integral equations; Conversion of integral equations into differential equations

Unit-V: Fredholm equations of second kind with separable kernels, Fredholm alternative theorem, Eigen values and eigen functions, Method of successive approximation for Fredholm and Volterra equations, Neumann's Series, Resolvent kernel.

Books recommended:

1. I.N. Sneddon: Elements of Partial Differential Equations, McGraw-Hill Pub.,1957.
2. T. Amaranath: An Elementary Course in Partial Differential Equations, Narosa Pub. 2005.
3. R.P. Kanwal: Linear Integral Equations, Birkhauser Verlag Pub.,1997.

4. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-IV(b): BASIC NUMBER THEORY

Unit-I: Fundamental theorem of arithmetic, Arithmetic functions, Dirichlet product of arithmetic functions, Ring of Arithmetic functions, Multiplicative arithmetic functions, Multiplicativity of $\sigma(n)$, $\tau(n)$, $\phi(n)$ and $\mu(n)$, Möbius inversion formula and its applications.

Unit-II: Quadratic residues, Euler's criterion, The Legendre symbol and its properties, Gauss lemma, Guass quadratic reciprocity law, Quadratic congruences with composite moduli.

Unit-III: Primitive roots, Structure of groups U_n of units modulo n, Existence of primitive roots, Representation of integers as sum of squares.

Unit-IV: Finite continued fractions, Infinite continued fractions, Approximations to irrational numbers, Periodic continued fractions, Pell's equation.

Unit-V: Algebraic Numbers, Algebraic number fields, Algebraic integers, Quadratic fields, Units and primes in quadratic fields, Euclidean quadratic fields, Primes in quadratic fields having unique factorization property.

Books Recommended:

1. D. M. Button, Elementary Number Theory, 7th edition, McGraw Hill Education, 2017
2. I. Niven, H. S. Zuckerman and H. L. Montgomery, An introduction to the Theory of Numbers, John Wiley and Sons, Inc., 2008.
3. Ramji Lal, Algebra I, Infosys Foundation Series in Mathematical Sciences, Springer, 2017.
4. K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory, ,GTM-84, Springer, 1990.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-V: Any one of the following

PAPER-V(a): PROJECT PRESENTATION

PAPER-IV(b): Computational Techniques Using C

M.A./M.Sc. SEMESTER II (YEAR I)

PAPER-I: ALGEBRA-II

Unit-I: Direct sums of modules, Free modules and vector spaces, Homomorphism extension property, Invariant dimension property, Left exactness of Hom functor, Projective modules.

Unit-II: Injective modules, Baer's characterization, Divisible groups, Existence of enough injectives, Submodules of finitely generated free modules over a PID, Torsion and torsion free modules, p-primary components, Cyclic modules.

Unit-III: Structure theorem for finitely generated modules over a PID, Elementary divisors and invariant factors, Direct sum decomposition of finite abelian groups into cyclic groups and their enumeration.

Unit-IV: Characteristic of a field, Field extensions, Algebraic and transcendental field extensions, Characterization and properties of algebraic extensions, Finitely generated extensions, Simple extensions, Straight edge and compass constructions, Constructible numbers.

Unit-V: Splitting field of a polynomial, Existence and uniqueness of splitting fields, Existence and uniqueness of finite fields, Structure of finite fields, Cycotomic polynomials and extensions, Separable and inseparable extensions, Perfect fields.

Books Recommended:

1. Ramji Lal, Algebra II, Infosys Foundation Series in Mathematical Sciences, Springer, 2017.
2. V. Sahai and V. Bist, Algebra, Narosa Publishing House, 2008.
3. D. S. Dummit and R. M. Foote, Abstract Algebra, Wiley, 2002.
4. J. A. Gallian, Contemporary Abstract Algebra, Cengage India Pv. Ltd., 2019
5. T. W. Hungerford, Algebra, Springer, 1974.
6. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-II: TOPOLOGY

Unit-I: Topological spaces, Metric topology, Open sets, Closed sets, Neighborhoods; Interior, closure, exterior, boundary, and limit points of a set, Dense sets, Separable spaces, Bases and Sub-bases, Subspaces and relative topology..

Unit-II: First and second countable spaces, Lindelöf spaces, Continuous maps and their characterizations, Open and closed maps, Homeomorphisms, Topological property, Product spaces, Projection maps, Convergence of nets and filters.

Unit-III: Quotient spaces and quotient maps, Separation axioms: T_0, T_1,T_2, Regular, T_3, Completely regular, normal, $T_{3\frac{1}{2}}$ and T_4 spaces, Their characterizations and basic properties, Statements of Urysohn's lemma, Tietze extension theorem, and Urysohn's metrization theorem.

Unit-IV: Compact spaces and their characterizations, Sequential and countable compactness, Characterization of compact metric spaces (limit point compactness, sequential compactness, complete and total boundedness), Local compactness, Statement of Tychonoff theorem.

Unit-V: Connected spaces, Connectedness of real line, Connected components, Path connected spaces, Totally disconnected spaces, Locally connected spaces, Locally path connected spaces, Path components, Properties of continuous functions on compact and connected spaces.

Books Recommended:

1. J.R. Munkers, Topology-A First Course, 2nd ed., Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
2. K.D. Joshi, Introduction to General Topology, NewAge International Publishers, New Delhi, 2000.
3. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Education, 2017.
4. T. B. Singh, Elements of Topology, CRC press, New Delhi, 2013.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-III: CLASSICAL MECHANICS

UNIT I: Dynamics of a system of particles:

The linear momentum and the angular momentum, Equations of motion, Conservation of linear and angular momentum, Motion of the centre of mass, Theorems on the rate of change of angular momentum about different points, Kinetic energy of a system of particles relative to the centre of mass of the system.

Rigid bodies as systems of particles, General displacement of a rigid body, Displacement of a rigid body about one of its points and the concept of angular velocity, Computation of the angular velocity of a rigid body in terms of the velocities of two particles of the system chosen appropriately.

UNIT II: Introduction to rigid body motion:

Moments and products of inertia about different axes of uniform rod, Rectangular lamina, Rectangular parallelopiped, Circular wire, Elliptical disc and sphere, Concepts of momental ellipsoid and principal axes.

The angular momentum and kinetic energy of a rigid body in terms of inertia constants, D'Alembert's principle, General equations of motion, Motion of a sphere on horizontal plane, Euler's dynamical equations of motion, Motion under no forces, Invariable line and invariable cone.

UNIT III: Lagrangian Formulation:

Generalized co-ordinates, Geometrical equations, Holonomic and non-holonomic systems, Configuration space, Lagrange's equations using D' Alembert's Principle for a holonomic conservative system, Deduction of equation of energy when the geometrical equations do not contain time *t* explicitly, Lagrange's multipliers case.

UNIT IV: Theory of small oscillations:

Theory of small oscillations, Lagrange's method, normal (principal) co-ordinates and the normal modes of oscillations, Small oscillations under holonomic constraints, Lagrange equations for impulsive motion.

UNIT V: Hamiltonian Formulation:

Generalized momentum and the Hamiltonian for a dynamical system, Hamilton's canonical equations of motion, Hamiltonian as a sum of kinetic and potential energies, Phase space and

Hamilton's variational principle, the principle of least action, Canonical transformations, Poisson-Brackets, Poisson-Jacobi identity, Hamilton-Jacobi theory (outline only).

Books Recommended:

1. Satya Deo and Ramij Rahaman, Classical Mechanics: An Introduction, Narosa Publishing House, New Delhi, 2022.
2. H. Goldstein, Classical Mechanics, Pearson,2011.
3. F. Chorlton, Text Book of Dynamics, CBS Publishers, New Delhi, 1999.
4. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-IV: Any one of the following

PAPER-IV(a): TENSORS AND RIMANNIAN GEOMETRY

Unit I: n-dimensional real vector space, Contravariant vectors, Dual vector space, Covariant vectors, Tensor product, Second order tensors, Tensors of type (r, s), Symmetry and skew symmetry of tensors, Fundamental algebraic operations, Inner product, Quotient law of tensors.

UNIT II: Riemannian metrics, Riemannian manifolds, examples, Affine connections, Covariant differentiation of tensor fields, Covariant derivative along a curve, Parallel transport, Levi-Civita connection, Fundamental Theorem of Riemannian Geometry.

UNIT III: Differential operator on Riemannian manifolds, Gradient vector fields, Divergence of a vector field, Laplacian operator, Lie derivatives of a tensor field with respect to a vector field.

UNIT IV: Riemannian curvature tensor, Identities satisfied by Riemannian curvature tensor, Sectional curvature , Schur's Theorem, Ricci curvature, Scalar curvature, Einstein manifolds, isometries, Notion of covering spaces.

. **UNIT V:** Length of a curve, Riemannian distance function, Geodesics, Local existence and uniqueness for geodesics, Exponential map, Gauss Lemma, Minimizing properties of geodesics, Geodesics normal coordinates.

Books Recommended:

1. R. S. Mishra, A course in Tensors with Application to Riemannian Geometry, Pothishala Pvt. Ltd. Allahabad, 1965.
2. M. P. do Carmo; Riemannian Geometry, Berkhauser, 1992.

3. P. Peterson; Riemannian Geometry, Springer, 2006.
4. J. M. Lee; Riemannian Manifolds: An Introduction to Curvature, Springer, 1997.
5. S. Gallot, D. Hullin. J. Lafontaine; Riemannian Geometry, Springer,3rd edition, 2004.
6. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-IV(b): HYDRODYNAMICS

UNIT I:

Real and ideal fluids, Lagrangian and Eulerian approaches, Convective transport of scalar and vector quantities, Differentiation following the motion and acceleration, Equation of continuity, Velocity potential, Body forces, Surface forces, Stress vector at a point, Nature of stresses, State of stress at a point, Stress tensor, Principal stresses and principal directions, Stress invariants.

UNIT II:

General displacement of a fluid element, Nature of strains, Rates of strain components, Relation between stress and rates of strain, Transformation of stress- components, Transformation of Rates of strain, Euler's equation of motion, Steady motion, Bernoulli's equation.

UNIT III:

Stream lines and vortex lines, Stream tubes and vortex tube, Helmholtz's vorticity theorem, Kelvin's circulation theorem, Energy flux, Mean potential over a spherical surface in a simply connected region, Kinetic energy in irrotational flow, Kelvin's minimum kinetic energy theorem, Uniqueness of the irrotational motion.

UNIT IV:

Two dimensional irrotational motion: Stream function, Complex potential, Concepts of line-sources, sinks, doublets and vortices, Superposition of solutions, The concept of images, The Vortex pair, Vortex rows: Single infinite row of line vortices, the Karman vortex street, Milne-Thomson Circle Theorem, Blasius Theorem, Complex potential for a uniform flow past a circular cylinder, Streaming and circulation about a fixed circular cylinder.

UNIT V:

Three dimensional irrotational flow, Concept of Sources, Sinks and doublets, Axisymmetric flows, Stokes stream function, Statements of Weiss's and Butler's sphere theorems and their applications, Liquid streaming past a stationary sphere, Uniform motion of a sphere in a liquid at

rest at infinity, Gravity waves – Surface waves on the infinite free surface of liquids, Waves at the interface between finitely and infinitely deep liquids.

Books Recommended:

1. L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Butterworth-Heinemann, 2nd Edition, 1987.
2. N. Curle and H. J. Davies, Modern Fluid Dynamics, Vol. I, D. van Nostrand Comp. Ltd., London, 1968.
3. Dr. M.D. Raisinghania, Fluid Dynamics, S. Chand & Company.
4. S. W. Yuan, Foundations of Fluid Mechanics, Prentice-Hall, Englewood Cliffs, NJ, 1967.
5. A. S. Ramsey, A Treatise on Hydrodynamics, Part I, G. Bell and Sons Ltd.
6. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-V: Any one of the following

PAPER-V(a): PROJECT PRESENTATION

PAPER-IV(b): Introduction to Latex

M.A./M.Sc. SEMESTER III (YEAR II)

PAPER-I: MEASURE AND INTEGRATION

Unit-I: Semi-algebra, Algebra, Monotone class, σ- algebra, Measure and outer measure, Measure spaces, Borel sets, Lebesgue outer measure on $\mathbb{R}$ and its properties, σ- algebra of Lebesgue measurable sets, Lebesgue measure space, Translation invariance of Lebesgue measure.

Unit-II: Outer and inner approximation of Lebesgue measurable sets, Vitali's theorem, The Cantor ternary set and Cantor-Lebesgue function, Continuity of measure, Caratheodory extension process of extending a measure on a semi-algebra to generated σ- algebra, Completion of a measure space.

Unit-III: Measurable functions on a measure space and their properties, Borel and Lebesgue measurable functions, Littlewood's three principle (statement only), Simple approximation Theorem, Simple functions and their integrals,

Unit-IV: Lebesgue integral on $\mathbb{R}$ and its properties, Comparison of Riemann integral and Lebesgue integral, Bounded convergence theorem, Integration of nonnegative measurable functions on a measure space, Fatou's lemma, Monotone convergence theorem, Integration of general measurable functions, Continuity of integration, Lebesgue dominated convergence theorem.

Unit-V: Inequalities of Young, Hölder and Minkowski, Normed Linear spaces, Riesz-Fischer theorem, Measure and integration on product spaces, Fubini's theorem, Lebesgue measure on $\mathbb{R}^2$ and its properties.

Books Recommended:

1. H.L. Royden, and P. M. Fitzpatrick, , Real Analysis, 4th Edition, Pearson, 2010.
2. I. K. Rana, An Introduction to Measure and Integration, 2nd edition, Narosa Publishing House India, 2000.
3. P. K. Jain and V. P. Gupta, Lebesgue Measure and Integration, New Age International (P) Limited, New Delhi, 1986.
4. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-II: ORDINARY DIFFERENTIAL EQUATIONS AND BOUNDARY VALUE PROBLEMS

UNIT I:

Initial value problems, Picard's iterations, Lipschitz conditions, Sufficient conditions for being Lipschitzian in terms of partial derivatives, Picard's theorem for local existence and uniqueness of solutions of an initial value problem of first order which is solved for the derivative, Uniqueness of solutions with a given slope, Singular solutions, *p*- and *c*-discriminant equations of a differential equation and its family of solutions respectively, Envelopes of one parameter family of curves, Singular solutions as envelopes of families of solution curves, Sufficient conditions for existence and nonexistence of singular solutions, examples.

UNIT II:

Linear independence and Wronskians, General solutions covering all solutions for homogeneous and non-homogeneous linear systems, Abel's formula, Method of variation of parameters for particular solutions, Linear systems with constant coefficients, Matrix methods, Different cases involving diagonalizable and non-diagonalizable coefficient matrices, Real solutions of systems with complex eigenvalues.

UNIT III:

Convergence of real power series, Radius and interval of convergence, Ordinary and singular points, Power series solutions, Frobenius' generalized power series method, Indicial equation, different cases involving roots of the indicial equation, Regular and logarithmic solutions near regular singular points.

UNIT IV:

Legendre's equation, Solution by power series method, polynomial solution, Legendre polynomial, Rodrigues' formula, Generating function, Recurrence relations, Orthogonality relations, Fourier-Legendre expansion, Bessel's equation, Bessel functions of I and II kind, Recurrence relations, Bessel functions of half-integral orders, Sturm comparison theorem, Zeros of Bessel functions, Orthogonality relations, Generating function.

UNIT V:

Boundary-value problems: Orthogonal and orthonormal sets of functions, Sturm-Liouville (S-L) problems, Eigenvalues and Eigenfunctions of S-L problems , Reality of eigenvalues and orthogonality of eigenfunctions of S-L problems, Singular Sturm-Liouville problems, Green's

function, Construction of Green functions, Solution of Boundary value problem by Green's function.

Books Recommended:

1. B. Rai, D. P. Choudhury and H. I. Freedman, A Course in Ordinary Differential Equations, Narosa Publishing House, New Delhi, 2002.
2. E. Kreyszig, Advanced Engineering Mathematics, Wiley India Pvt. Ltd., 8thEdition, 2001.
3. S.G. Deo, V. Raghavendra, R. Kar and V. Lakshmikanthanm, text Book of Ordinary Differential equations, McGraw Hill Education, 2017.
4. E. A. Coddington, An Introduction to Ordinary Differential Equations, Prentice Hall of India, New Delhi, 1968.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-III: DIFFERENTIAL GEOMETRY OF MANIFOLDS

UNIT I: Topological manifolds, Compatible charts, Smooth manifolds, examples, Smooth maps and diffeomorphisms, Definition of a Lie group, examples.

UNIT II: Tangent and cotangent spaces to a manifold, Derivative of a smooth map, Immersions and submersions, submanifolds , Vector fields, Algebra of vector fields, φ-related vector fields, Left and right invariant vector fields on Lie groups.

UNIT III: Integral curves of smooth vector fields, Complete vector fields, Flow of a vector field, Distributions, Tensor fields on manifolds, r-forms, Exterior product, Exterior differentiation, Pull-back differential forms.

UNIT IV: Affine connections (covariant differentiation) on a smooth manifold, Torsion and curvature tensors of an affine connection, Identities satisfied by curvature tensor.

UNIT V: Riemannian metrics, Riemannian manifolds, Submanifolds, Local isometry and isometry, Levi-Civita connection, Fundamental Theorem of Riemannian Geometry, Riemannian curvature tansor, Identities satisfied by Riemannian curvature tensor, Ricci tensor, Scalar curvature, Sectional curvature of Riemannian manifolds.

Books Recommended:

1. S. Kumaresan; A course in Differential Geometry and Lie groups, Hindustan Book Agency, 2002.

2. U. C. De, A. A. Sheikh; Differential Geometry of Manifolds, Narosa Publishing House, 2007.
3. W. M. Boothby; An Introduction to Differentiable Manifolds and Riemannian Geometry, Academic Press, revised, 2003.
4. T. J. Willmore; Riemannian geometry, Oxford Science Publication, 1993.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-IV: Any one of the following:

PAPER-IV(a): MATHEMATICAL MODELLING

Unit I: Simple situations requiring mathematical modelling, Techniques of mathematical modelling, Classifications, Characteristics and limitations of mathematical models, Some simple illustrations.

Unit II: Mathematical modelling through differential equations: Overview of basic concepts in ODE and stability of solutions, Critical points and their local and global stability, Stability by variational matrix method and Liapunov's direct method.

Unit III: Linear growth and decay models, Nonlinear growth and decay models, Mathematical modelling in dynamics through ordinary differential equations of first order, Single species population model: The exponential model and the Logistic model.

Unit IV: Biological interactions, Models for interacting species, Lotka-Volterra predation models, Dynamics of Simple Lotka-Volterra model, Role of density dependence in the prey, Analysis of a Predator-Prey Model with limit cycle periodic behaviuor.

Unit V: Competition: Lotka-Volterra models, Extension to Lotka-Volterra models, Competition for fixed resources and renewable resources, Models for Mutualism, Obligate and non-obligate mutualism.

Books Recommended:

1. J. N. Kapur, Mathematical Modeling, New Age International,1988.
2. J. D. Murray, Mathematical Biology: An Introduction, Springer, 2002 .
3. H.I. Freedman, Deterministic Mathematical Models in Population Ecology, Marcel-Dekker, New Yark, 1980 .

4. G, F, Simmons, Differential Equations with Applications and Historical Notes, Tata-McGraw Hill 1991.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-IV(b): ADVANCED LINEAR ALGEBRA

Unit-I: Algebraic and geometric multiplicities of eigenvalues, Invariant subspaces, Minimal polynomial, Triangulable linear operators, Characterizations of diagonalizable linear operators and matrices in terms of multiplicities and minimal polynomial.

Unit-II: Simultaneous triangulation and diagonalization, Invariant direct sum decompositions, Primary decomposition theorem, Characterization of diagonalizability in terms of projections, Diagonalizable and nilpotent parts of an operator.

Unit-III: Cyclic subspaces and annihilators, T-annihilator, cyclic decompositions and rational canonical form.

Unit-IV: The Jordan canonical form, Symmetric bilinear forms, Diagonalization of symmetric matrices, Sylvester's law of inertia.

Unit-V: Positive definite matrices and polar decomposition, QR, LU and Cholesky decomposition of matrices, Singular value decomposition.

Books Recommended:

1. K. Hoffman and R. Kunze, Linear Algebra, PHI, 2015.
2. V. Sahai and V. Bist, LinearAlgebra, Narosa Publishing House, 2013.
3. Ramji Lal, Algebra II, Infosys Foundation Series in Mathematical Sciences, Springer, 2017.
4. S.H. Friedberg, A. J. Insel and L. E. Spence, Linear Algebra, PHI, 2003.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-V: Any one of the following:

PAPER-V(a): Project Presentation Based On Survey/ Seminar/ Assignment

PAPER-IV(b): Introduction to SCILAB/MATLAB

M.A./M.Sc. SEMESTER IV (YEAR II)

PAPER-I: FUNCTIONAL ANALYSIS

Unit-I: Normed Linear spaces, Equivalent norms, Complete metric spaces, Banach Spaces, Examples and their topological properties, Finite dimensional normed linear spaces, Holder's and Minkowski's inequalities, l^p-spaces, L^p-spaces, Subspace and quotient space of Banach spaces.

Unit-II: Continuity and Convergence, Continuous linear transformations, Bounded linear transformations, Normed linear space of bounded linear transformations, Baire category theorem, Linear functionals, Hahn-Banach theorem,

Unit-III: Natural embedding of N into N^{**}, Conjugate spaces, Reflexivity, Weak and Weak*-topology on a conjugate space, Open mapping theorem, The Closed graph theorem, Uniform boundedness theorem, Conjugate of an operator.

Unit-IV: Inner product spaces, Cauchy-Schwarz inequality, Hilbert Spaces, Orthogonal complements, Orthonormal sets, Complete orthonormal sets, Bessel's inequality, Fourier expansion, Parseval's identity, Gram Schmidt orthogonalization process.

Unit-V: Riesz representation theorem, Reflexivity of Hilbert spaces, Adjoint of an operator, Self adjoint operators, Normal operators, Unitary operators, Projections, Spectrum of an operator, Spectral theorem for a normal operator on a finite dimensional Hilbert space.

Books Recommended:

1. S. Ponnusamy, Foundation of Functional Analysis, Narosa Publishing House, New Delhi,2002.
2. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Education, 2017.
3. E. Kreyszig: Introductory Functional Analysis with Applications: Wiley student Edition 2007.
4. J. B. Conway: A course in Functional Analysis, Springer: Second Edition, 2007.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-II: ADVANCED FLUID MECHANICS

UNIT-I: Stress Principle of Cauchy, Equations for conservation of linear and angular Momentum, Constitutive equations for Newtonian fluids, Newton's law of viscosity, Navier-Stokes equations in Vector and Tensor forms, Navier- Stokes equations in orthogonal coordinate systems (particularly in Cartesian, cylindrical and spherical coordinate systems).

UNIT-II: Vorticity equations; Energy dissipation due to viscosity, Dynamical similarity and dimensionless numbers and their significance in the fluid dynamics, Some exact solutions –Fully developed plane Poiseuille and Couette flows between parallel plates, Steady flow between pipes of uniform cross-section.

UNIT-III: Couette flow between coaxial rotating cylinders, Small Reynolds number flow – Flow between steadily rotating spheres, Stokes equations, Dynamic equation satisfied by stream function, Relation between pressure and stream function; General stream function solution of Stokes equations in spherical polar coordinates; Steady flow past a sphere, Drag on a body.

UNIT-IV: Flow past a circular cylinder, Stokes paradox, Boundary layer concept, Two-dimensional boundary layer equations, Boundary layer on a semi-infinite plane, Blasius equation and solution, Karman's Integral method, Displacement thickness, Momentum thickness and Energy thickness.

Unit-V: Nature of Magnetohydrodynamics, Main assumptions of MHD, Basic equations of non- viscous and viscous magnetohydrodynamics: mass, momentum and energy conservation laws. Basic Properties of the magnetic field and MHD terms: Magnetic Reynolds number, magnetic viscosity, magnetic pressure, magnetic diffusion and frozen- in- effect. Magnetohydrodynamic boundary conditions, Magnetohydrodynamic Flows, Formulation and solution of Linear flow.

Reference books:

1. Z.U.A. Warsi, Fluid Dynamics, CRC Press (2005)
2. J. Happel and H. Brenner, Low Reynolds Number Hydrodynamics, Kluwer Academic Publishers group, (1983).

3. T.C. Papanastasiou, G.C.Georgiou, A.N.Alexandrou, Viscous Fluid Flow; CRC Press (2000).
4. L. D. Landau and E. M. Lifshitz, Classical Electrodynamics, Butterworth-Heinemann, 2nd Edition, 1984.
5. A. Jaffery, Magnetohydrodynamics, Oliver and Boyd, N.Y. 1966.
6. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-IV: Any one of the following

PAPER-IV(a): WAVELETS

UNIT I: The discrete Fourier transform and the inverse discrete Fourier transform, their basic properties and computations, The fast Fourier transform, Construction of wavelets on Z_N, First stage and by iteration, The Haar system, Shannon wavelets, Daubechies' D6 wavelets on Z_N, Description of $l^2(Z)$, $L^2[-\pi,\pi], L^2(R)$, Their orthonormal bases,

UNIT II: Fourier transform and convolution on $l^2(Z)$, wavelets on Z ,Haar wavelets on Z, Daubechies' D6 wavelets for $l^2(Z)$.Orthonormal bases generated by a single function in $L^2(R)$, Fourier transform and inverse Fourier transform of a function f in $L^1(R)\cap L^2(R)$, Parseval's relation, Plancherel's formula, Orthonormal wavelets in $L^2(R)$, Balian-Low theorem.

UNIT III: Multi-resolution analysis and MRA wavelets, certain function in $L^2(R)$ for which $\{\psi_j, k\}$ does not form an orthonormal system, compactly supported wavelets, Band-limited wavelets.

UNIT IV: Franklin wavelets on R, Characterization of MRA wavelets (Sketch of the proof), Minimally Supported Wavelets, Wavelet Sets, Characterization of two-interval wavelet sets, Shannon wavelet, Journe's wavelet.

Books Recommended:

1. Michael W. Frazier, An Introduction to Wavelets through Linear Algebra, Springer Verlag, 1999.
2. Eugenio Hernández and Guido Weiss, A First Course on Wavelets, CRC Press, 1996.
3. G. Kaiser, A Friendly Guide to Wavelets, Birkhauser, 1994.

4. C. K. Chui, An Introduction to Wavelets, Academic Press, 1992.
5. Suggested digital platforms: NPTEL/SWAYAM/MOOCS.

PAPER-IV(b): REPRESENTATION THEORY OF FINITE GROUPS

Unit-I: Simple modules, Schur's lemma, Semi-simple rings and modules, Wedderburn structure theorem for semi-simple modules and rings.

Unit-II: Group algebras, Maschke's theorem, Linear and matrix representation of a finte group, Representations of G and k(G)-modules, Equivalent representations, Decomposition of regular representation, Number of irreducible representation.

Unit-III: Characters, Irreducible characters, Orthogonality relations, Integrality properties of character, Character ring, Burnside's p^aq^b theorem.

Unit-IV: Representations of direct product of two groups, Induced representation, The character of an induced representation, Frobenius reciprocity theorem, Construction of irreducible representation of D_n, A_4, S_4 and S_5.

Books Recommended:

1. Ramji Lal, Algebra II, Infosys Foundation Series in Mathematical Sciences, Springer, 2017.
2. D. S. Dummit and R. M. Foote, Abstract Algebra, Wiley, 2002.
3. J. P. Serre, Linear Representations of finite Groups, GTM-42, Springer, 1977.
4. L. Dornhoff, Group Representation Theory , Part A, Academic Press, 1971.

PAPER-IV(c): ALGEBRAIC NUMBER THEORY

Unit-I: Algebraic number fields, Algebraic numbers and algebraic integers, Ring of algebraic integers, Calculation for quadratic, cubic and cyclotomic case.

Unit-II: Norm and traces, Integral bases and discriminants, Ideals in $O_{K,}$ Dedekind domains, Fractional ideals and unique factorization.

Unit-III: Dedekind's theorem, Factorization in O_K, The ideal class group, Lattices in $\mathbb{R}^n$, Minkowaski's bound, Finiteness of ideal class group.

Unit-IV: Exponents of ideal class groups. Dirichlet's unit theorem, Units in real quadratic real fields and cyclotomic fields.

Books Recommended:

1. J. Esmonde and M. R. Murty, Problems in Algebraic Number Theory, Springer, 1999.
2. J. Marcus, Number Fields, Springer, 1977.
3. K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory, ,GTM-84, Springer, 1990.

PAPER-IV(d): SPECIAL FUNCTIONS

Unit-I: The Gamma Function**:** Analytic Character, Tannery's theorem, Euler's limit formula, Duplication formula, Eulerian integral of the first kind, Euler's Constant, Canonical product, Asymptotic expansions, Watson's lemma, Hankel's contour integral.

Unit-II: The Hypergeometric Function: An Integral representation, Its differential equation and solutions, F(a, b, c; 1) as a function of the parameters, evaluation of F(a, b, c; 1), contiguous function relations, the hypergeometric differential equation, logarithmic solutions of the hypergeometric equation.

Unit-III: F(a, b, c; z) as a function of its parameters, Elementary series manipulations, Simple transformations, relations between functions of (z) and, (1-z) quadratic transformations, theorem, due to Kummer, additional properties, Confluent hypergeometric function: Basic properties of ${}_1F_1$, Kummers first formula, Kummers second formula.

Unit-IV: Generalized hypergeometric series: The function pFq, the exponential and bionomial functions, differential equation, contiguous function relations, integral representation pFq, with unit argument, Saalshutz, theorem, Whipples theorem, Dixons theorem, Barnes's contour integral of F(a, b, c; z).

Books Recommended:

1. N.N. Lebedev and R. A. Silverman, Special Functions and their applications, Dover Publication Inc.1972.
2. A. Chakrabarti, Elements of Ordinary Differential Equations and Special Functions, New Age International Publisher, 1996.
3. Earl. D. Ranvillie, Special Functions, Macmillan, 1960.

PAPER-IV(e): GALOIS THEORY

Unit-I: Field extensions, Algebraic extensions, Splitting field of a polynomial, Separable and inseparable extensions, Primitive element theorem.

Unit-II: Automorphism of fields, Dedekind's theorem, Fixed fields, Normal extensions, Splitting fields and normality, Normal closures.

Unit-III: Galois extensions, Fundamental theorem of Galois theory, Computation of Galois groups of polynomials.

Unit-IV: Cyclic extensions, Radical extensions, Galois' criterion for solvability of polynomial by radical operations, Abel-Ruffini theorem.

Books Recommended:

1. Ramji Lal, Algebra II, Infosys Foundation Series in Mathematical Sciences, Springer, 2017.
2. V. Sahai and V. Bist, Algebra, Narosa Publishing House, 2008.
3. D. S. Dummit and R. M. Foote, Abstract Algebra, Wiley, 2002.
4. P. Morandi, Field and Galois Theory, springer 1996.

PAPER-IV: MAJOR RESEARCH PROJECT/DESSERTATION